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Prediction of flexural fatigue life and failure probability of normal weight concrete

K.M.A. Sohel✉, M.H.S. Al-Hinai, A. Alnuaimi, M. Al-Shahri, S. El-Gamal

Department of Civil and Architectural Engineering, Sultan Qaboos University, (Muscat, Oman)
✉: kmasohel@squ.edu.om

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ABSTRACT: Fatigue life has to be considered in the design of many concrete structures at various stress levels and stress ratios. Many flexural fatigue test results of plain normal-weight concrete are available in the literature and almost every set of test results provides different fatigue equations. It is necessary, though, to have a common fatigue equation to predict the design fatigue life of concrete structures under flexural load with reasonable accuracy. Therefore, a database of flexural fatigue test results was created for concrete with strengths ranging from 25 to 65 MPa; this database was used to derive new fatigue equations (Wöhler fatigue equation and *S–N* power relationship) for predicting the flexural fatigue life of normal-weight concrete. The concept of equivalent fatigue life was introduced to obtain a fatigue equation using the same stress ratio. A probabilistic analysis was also carried out to develop flexural fatigue equations that incorporate failure probabilities.

KEY WORDS: Concrete; Fatigue; Flexural strength; Durability; Mechanical properties.

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RESUMEN: *Predicción de la vida a fatiga por flexión y la probabilidad de falla del hormigón de densidad convencional.* La vida a fatiga debe ser considerada en el diseño de muchas estructuras de hormigón bajo varios niveles de tensión y relaciones de tensión. Muchos resultados de pruebas de fatiga por flexión del hormigón convencional (densidad normal) están disponibles en la literatura y casi todos proporcionan diferentes ecuaciones de fatiga. Sin embargo, es necesario tener una ecuación de fatiga común para predecir la vida a fatiga de diseño de las estructuras de hormigón bajo carga de flexión con una precisión razonable. Por lo tanto, se creó una base de datos de resultados de ensayos de fatiga por flexión para hormigón con resistencias que oscilan entre 25 y 65 MPa; esta base de datos se utilizó para generar nuevas ecuaciones de fatiga (ecuación de fatiga de Wöhler y relación de potencia S–N) para predecir la vida a fatiga por flexión del hormigón de densidad normal. El concepto de vida a fatiga equivalente se introdujo para obtener una ecuación de fatiga utilizando la misma relación de tensión. También se llevó a cabo un análisis probabilístico para desarrollar ecuaciones de fatiga por flexión que incorporen probabilidades de falla.

PALABRAS CLAVE: Hormigón; Fatiga; Resistencia a flexión; Durabilidad; Propiedades mecánicas.

1. INTRODUCTION

Many concrete structures such as bridge decks, airport runways, concrete pavements, and offshore structures experience a million cycles of repetitive loading in their service life (1, 2). The exposure to repetitive loading reduces the stiffness of the concrete structures; this in turn leads to fracture generation as a result of changes caused by the progressive growth of micro-cracks. When the repeated loads are applied at high frequency for a prolonged period, this can eventually lead to fatigue failure (1–3). For this reason, in the design of these concrete structures, increasing attention is now being paid to the fatigue characteristics of the constituent materials.

Concrete is a compound heterogeneous construction material that consists mainly of cement, water, fine aggregates, and coarse aggregates. It contains both enormous micro-sized capillary pores and millimeter-sized air voids that result from the hydration process, shrinkage, and other causes. These imperfections in the concrete make it highly susceptible to repeated or cyclic loads. Repetitive loading causes progressive and permanent internal structural changes, known as a fatigue process that leads to failure. Therefore, considerable research effort has gone into understanding the behavior of concrete under cyclic loading (1–4). There are several other motivations for these research efforts to study the fatigue behavior of concrete. Two of these are the increasing use of plain concrete at airfields and the concrete pavements of highways being subjected to repeated loads. This has necessitated the design of these structures to accommodate cyclic loading. Secondly, the introduction of new types of concrete with high strength characteristics requires them to perform satisfactorily under high and repeated loading. For these reasons, the study of concrete behavior under varying loads is more critical than that of concrete in a static state. Moreover, it is important to identify the effects of repeated loading on the material characteristics of concrete (such as the static strength, durability, and stiffness) that could be significantly reduced even if the repeated loading does not lead to fatigue failure. Furthermore, understanding the causes and nature of fatigue is very important from both an economic and a structural safety point of view.

The study of flexural fatigue effects on concrete structures began as early as 1906 with a study by Feret (5). The fatigue tests for concrete are usually carried out by applying sinusoidal wave loading to a specimen. The sinusoidal waves include both the maximum amplitude (S_{max}) and the minimum amplitude (S_{min}) of the applied load in addition to the cyclic loading frequency and the maximum number of cycles (N). Most of the studies focused on establishing a relationship between the applied stress level ($S = S_{max}/f_r$) and the number of loading cycles (N) to failure, where f_r is the static flexural strength (modulus of rupture) of the concrete (6–8). The established relationship is known as the S–N curve or the Wöhler fatigue curve (6, 7). The equation of this curve is known as the Wöhler fatigue equation, which is expressed by Equation [1]:

$$S = S_{max}/f_r = a + b\log(N) \qquad [1]$$

where a and b are the coefficients that can be obtained by a linear regression analysis of the plotted test fatigue lives. Equation [1] shows the relationship between the applied stress level (S) and the number of cycles until failure on a logarithmic scale ($\log(N)$). Therefore, this equation is also known as the single-logarithm fatigue equation (8).

Owing to the widely scattered nature of concrete fatigue test results, it is important to test many specimens at the same stress level (S) with the same rate and period. In addition, it is essential to implement the probabilistic approach to ensure the reliability of the fatigue test data and to secure the concrete structures against fatigue failure. In the literature (6, 8), many mathematical models are to be found that apply the probabilistic approach to the fatigue test data of cementitious materials. Log-normal distribution and three- or two-parameter Weibull distribution are some of the models used to represent the fatigue test data statistically.

It is well established that members designed to resist a specific static loading will not endure if the same load is applied to them cyclically. Therefore, acquiring knowledge of the fatigue life of concrete is important in order to ensure an efficient, safe, and economical structural design and also to provide a basis for understanding the fatigue parameters and predicting the fatigue-life distribution. It has also been found that fatigue equations obtained from different test data sets differ from one another and provide different estimations of fatigue life (9). Therefore, all the relevant test data should be combined for the purposes of generating a general fatigue equation that can be used in the design.

One of the main objectives of this study is to determine the relationship between fatigue life and stress level for plain normal-weight concrete (NWC) using a large number of the relevant fatigue test data available in the published literature. Another objective is to perform statistical analyses on the equivalent fatigue test data to show the fatigue-life distribution of NWC for different stress ratios (R=S_{min}/S_{max}) at various stress levels (S).

2. FLEXURAL FATIGUE LIFE OF NORMAL-WEIGHT CONCRETE

For the purposes of this research, the available flexural fatigue test data for plain NWC with different compressive strengths have been collected from the published literature. A total of 465 flexural fatigue test results for NWC with a compressive strength ranging from 27 MPa to 62.3 MPa were collected and used in the analysis. For all the collected test results, the loading frequency was in the range of 1–20 Hz. The collected flexural fatigue test data for plain NWC, as found in the literature, are listed in Appendix A.

Many studies on the fatigue of concrete have concluded that a loading frequency in the range of 1–20 Hz had no effect on the fatigue life of plain concrete (4, 10). Therefore, in this research, only the concrete compressive strength and the applied stress ratio (*R*) were taken into account as effective variables for the fatigue strength analysis of the plain NWC.

2.1 Equivalent fatigue life (EN)

The results of 16 different experimental tests for flexural fatigue were used in the analysis. Since the fatigue test data have different stress levels ($S = S_{max}/f_r$) and different stress ratios ($R = S_{min}/S_{max}$), it is not appropriate to use them directly for the purposes of comparison. Furthermore, it is difficult to perform a direct statistical analysis of the fatigue-life data using both variable stress level (*S*) and stress ratio (*R*). For this reason, the analysis was conducted for one variable at a time using the equivalent fatigue-life (*EN*) concept, which was first used by Shi *et al.* (8), as given by Equation [2]. According to this concept, all the data concerning fatigue life at a specific stress level (*S*) with different stress ratios (*R*) can be transferred to get a common equivalent fatigue life (*EN*). Since it is well established that the fatigue life (*N*) increases as the stress ratio (*R*) increases (8, 11), the test data with different stress ratios were converted to the equivalent fatigue-life (EN) using Equation [2]:

$$EN = N^{1-R} \qquad [2]$$

where *EN* is the equivalent fatigue life; *N* is the test fatigue life; *R* is the stress ratio. Equation [2] can also be used to transform the *EN* into a fatigue life (*N*) with a specific *R*-value. As most of the fatigue test data relate to the stress ratio $R = 0.1$, all the other test data were transformed from *EN* to fatigue lives (*N*) with a stress ratio $R = 0.1$, using Equation [2] at each stress level (*S*). By definition, the *EN* corresponds to the fatigue life for the stress ratio $R = 0$.

The fatigue lives are determined against the applied stress level (*S*), which is a dimensionless quantity since it is a ratio of the maximum applied stress (S_{max}) to the modulus of rupture (f_r), that is, (S_{max}/f_r). This was done to eliminate the effects of concrete strength and the water–cement ratio, the type and gradation of aggregates, and the type and amount of cement on fatigue life *N* (8, 12). It is also well established that the reliability of the fatigue equation depends on the number of test data. Based on the above discussion, a large number of relevant fatigue test data were used to generate one fatigue equation for NWC which can be used to design concrete structures that are able to withstand being subjected to repetitive loading. Because this study combines a large number of relevant test data from various sources, the fatigue lives at the same stress level are highly scattered. A statistical procedure known as Chauvenet's criterion was employed to eliminate some of the outlier data from the fatigue data set (13, 14).

2.2 Wöhler fatigue equation for normal-weight concrete

The equivalent fatigue lives were used to obtain the *S*–*N* curves for the stress ratios $R = 0$ and $R = 0.1$. Figure 1 shows the plot of *S*–*N* curves using the 16 flexural fatigue-life data of NWC. A linear regression analysis based on the best-fit curve was performed. From the equations of the best-fit curves in Figures 1(a) and (b), the values of the coefficients (*a* and *b*) of the Wöhler fatigue equation (Equation [1]) were obtained for the stress ratios $R = 0$ and $R = 0.1$. The correlation coefficient (C_c) of the regression analysis is 0.87 for both stress ratios, as shown in Figure 1. This low value of the C_c (less than 0.9) indicates the scatter characteristics of the fatigue test data. However, the value of $C_c > 0.70$ for the plotted data refers to a strong relationship between *S* and log(*N*) (15). For both stress ratios ($R = 0$ and $R = 0.1$), the obtained values of the coefficients (*a* and *b*) of the Wöhler fatigue equation for the fatigue lives of NWC are given in Table 1.

Using these coefficients, the Wöhler fatigue equations for the plain NWC were generated; these are presented and discussed in section 5.1. From these equations, the fatigue strengths of the plain NWC for any desired number of cycles can be calculated.

TABLE 1. Coefficients *a* and *b* of Wöhler fatigue equation for plain NWC.

Stress ratio (*R*)	Coefficient of Wöhler fatigue equation	
	a	*b*
0.0	1.0367	–0.0758
0.1	1.0367	–0.0682

2.3 Power relationship of fatigue life (double-logarithm fatigue equation)

The power relationship of fatigue life was mainly developed for concrete pavements to estimate the

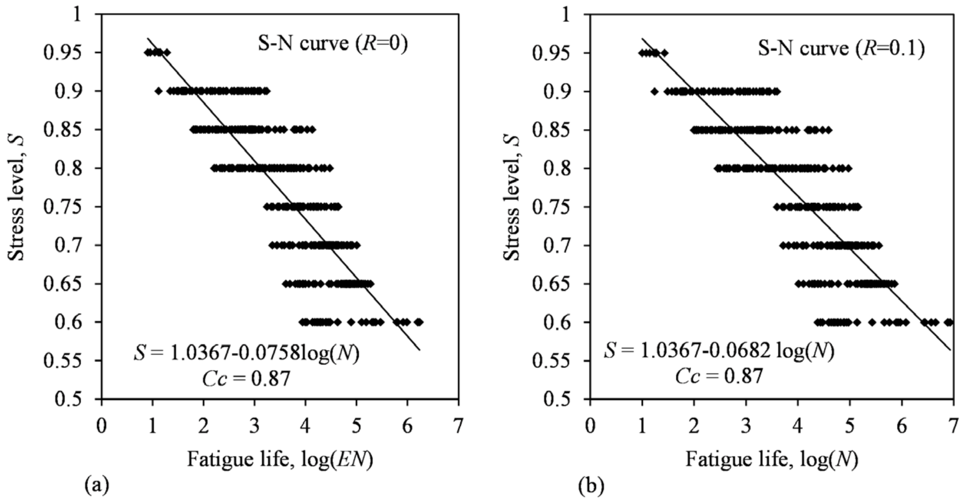

FIGURE 1. *S–N* curves (Wöhler fatigue curves) of NWC for (a) $R = 0$ and (b) $R = 0.1$.

flexural fatigue life of concrete (16). It relates the dimensionless stress level ($S=S_{max}/f_r$) to the number of loading cycles (N) given by Equation [3] (5, 6, 17, 18):

$$N(S)^m = C \qquad [3]$$

where C and m are the empirical constants, N is the fatigue life, and S is the applied stress level. This equation has wide applicability since the stress level $S = S_{max}/f_r$ is expressed as a dimensionless form. Here, S_{max} is the maximum applied stress and f_r is the concrete modulus of ruptures (flexural strength). Taking the logarithm of both sides of Equation [3], the following expression can be obtained (Equation [4]):

$$\log(N) = \log(C) - m\log(S) \qquad [4]$$

The values of C and m can be determined from the plot of log(N) versus log(S), as shown in Figure 2. For instance, from Figure 2(a), the values of m and C are 16.382 and 42.20 respectively for the stress ratio R=0. The correlation coefficient (C_c) of the regression analysis is 0.86 for both stress ratios, as shown in Figure 2. The empirical constants (m and C) obtained for different stress ratios are summarized in Table 2.

TABLE 2. Parameters of the power relationship of the equivalent fatigue life.

R	*m*	*C*
0.0	16.3820	42.20
0.1	18.0202	63.96

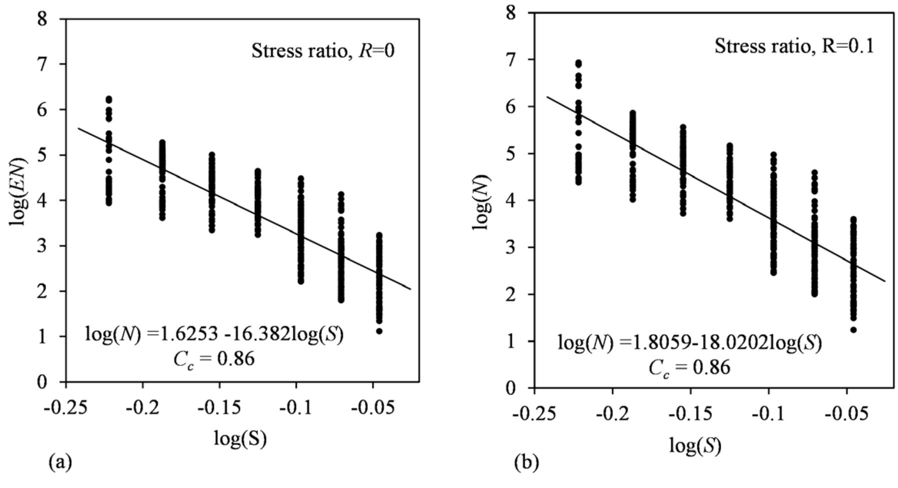

FIGURE 2. Estimating the empirical constants of *S–N* power relationships for stress ratio R: (a) $R = 0$ and (b) $R = 0.1$.

3. PROBABILISTIC ANALYSIS OF FATIGUE-LIFE DATA

The statistical nature of the fatigue test data exhibits a larger scatter than that of the static test data. The statistical variability may arise from the variation of a number of design factors such as the applied load and the heterogenetic nature of the material, which lead to an increase in the uncertainties in the design. Therefore, applying probabilistic analysis to the fatigue test data renders it more realistic and provides adequate resistance to the fatigue failure of concrete structures.

Numerous mathematical probability models have been developed to represent the probabilistic distribution of concrete fatigue-life data. Weibull distribution and Gumbel (19) distribution are some of the models used in many studies to represent the fatigue test data statistically. However, Gumbel distribution is generally used for the extreme values from some sets of fatigue data. On the other hand, Weibull distribution is widely used for both concrete and metal fatigue analysis to find out a mathematical model for the prediction of fatigue-life for a certain percentage of failure probability. Many studies (3, 6, 8, 20, 21) have shown that the two-parameter Weibull distribution can be used to describe the distribution of the fatigue-life data of cementitious materials, since it provides safer and greater reliability, as proved both statistically and experimentally. Therefore, the two-parameter Weibull distribution was used in this study to describe the probabilistic distribution of the flexural fatigue-life of the NWC. The failure probability function or the cumulative distribution function (CDF) of the two-parameter Weibull distribution is expressed as follows (Equation [5]) (6, 8, 22):

$$P_f(N) = 1 - e^{\left[-\left(\frac{n}{u}\right)^{\alpha}\right]};\ n > 0 \qquad [5]$$

where $P_f(N)$ is the failure probability function, n is the specific fatigue life of concrete at a particular stress level S, u is the scale parameter, and α is the shape parameter or the Weibull slope at the stress level S.

The survival probability function (L_n) or the reliability function of the Weibull distribution can be expressed by Equation [6] as follows (6, 8, 22, 23):

$$L_n = 1 - P_f(N) = e^{\left[-\left(\frac{n}{u}\right)^{\alpha}\right]};\ n > 0 \qquad [6]$$

where L_n is the confidence or survival probability function, n is the fatigue life, and α and u are the shape and scale parameters respectively of the Weibull distribution. The graphical, moment and maximum likelihood methods have been used by several researchers (3, 6, 8, 22) to obtain the values of the shape parameter (α) and the scale parameter (u) of the Weibull distribution at each stress level (S).

Other methods, such as the S–N power relationship, can be used to obtain a single value of the shape parameter (α) for all stress levels (3, 6, 24), whereas the scale parameter (u) is different at each stress level. This method is based on the approximate assumption of constant variance for all stress levels. This method was applied to the fatigue-life data for both stress ratios (R=0 and R=0.1), and is discussed in detail in the following sections.

3.1 Estimating Weibull distribution parameters using S–N relationship

As mentioned in the previous section, the S and N can be related by means of a power equation, as shown in Equations [3] and [4]. Equation [4] can be rewritten in a linear relationship format as given in Equation [7]:

$$Y = A + BX \qquad [7]$$

where $A = \log(C)$, $B = -m$, $X = \log(S)$, and $Y = \log(N)$

As mentioned in the previous section, the concrete fatigue life (N) follows the Weibull distribution (25). The standard deviation of log (N) and the Weibull distribution parameter (α) are interrelated, as presented in Equation [8]:

$$\sigma = \frac{\pi}{\alpha\sqrt{6}} \qquad [8]$$

where σ is the constant standard deviation of $Y = \log(N)$ in Equation [7] for all stress levels (S). The Weibull distribution parameter u may be obtained from the expression in Equation [9] (6, 26):

$$\ln(u) = \left(\frac{0.5772}{\alpha}\right) + \ln\left[C\left(\frac{S_{max}}{f_r}\right)^{-m}\right] \qquad [9]$$

The values of C and m can be determined from the plot of X and Y in Equation [7]. The values of C and m obtained for NWC at different stress ratios are given in Table 3. The calculated values of σ for the stress ratios R = 0 and R = 0.1 are 1.108 and 1.229, respectively. Thus, the estimated values of the shape parameter (α) of the Weibull distribution are 1.158 for the stress ratio R = 0 and 1.044 for the stress ratio R = 0.1. The values of the scale parameters u of the Weibull distribution for each stress level (S) were estimated using Equation [9], as presented in Table 3.

It can be seen in Table 3 that for all the stress levels there is only one value for the shape parameter α of the Weibull distribution, whereas the scale parameter (u) of the Weibull distribution was calculated separately for each stress level (S).

TABLE 3. Weibull distribution parameters (α and *u*) using the *S–N* relationship method for various stress levels (*S*) and stress ratios (*R*).

S	R = 0		R = 0.1	
	α	u	α	u
0.90	1.325	367	1.126	713
0.85	1.325	935	1.126	1997
0.80	1.325	2524	1.126	5954
0.75	1.325	7266	1.126	19049
0.70	1.325	22499	1.126	66041
0.65	1.325	75757	1.126	251066
0.60	1.325	281124	1.126	1062203

4. DESIGN FATIGUE LIFE AND FAILURE PROBABILITY

The fatigue-life data of the plain NWC used in this analysis showed a large scatter at a given stress level due to the heterogeneity and inherent material variability of concrete. Therefore, a design fatigue life should be selected with an acceptable failure probability. As discussed above, the equivalent fatigue lives and the fatigue lives with R = 0.1 follow the two-parameter Weibull distribution at all stress levels (S). Therefore, the design fatigue life (N_D) with different failure probabilities can be calculated using the Weibull distribution function. The design fatigue life (N_D) which incorporates an acceptable failure probability (P_f) at a specific stress level (S) can be obtained by rewriting Equation [5], as shown in Equation [10].

$$N_D = u\left[\ln\left(\frac{1}{1-P_f}\right)\right]^{\frac{1}{\alpha}} \qquad [10]$$

The design flexural fatigue lives (N_D) for NWC at different stress levels (S) with the corresponding failure probabilities (P_f) of 0.01, 0.05, 0.10, 0.2, 0.25, and 0.50 were accordingly calculated using Equation [10]. The corresponding α and u values for different stress levels (S) as given in Table 3 were used to calculate the N_D for two different stress ratios. The calculated design fatigue lives (N_D) are presented in Tables 4 and 5 below for the stress ratios R = 0 and R = 0.1 respectively. In addition, the design fatigue lives calculated with different failure probabilities are plotted in Figures 3 and 4 below for the stress ratios R = 0.0 and R = 0.1 respectively. These S–N–P_f curves describe the relationship between the stress level (S), the design fatigue life (N_D) and the failure probability (P_f). Regression analyses were performed to obtain the equation of each curve.

TABLE 4. Design fatigue life (N_D) for the stress ratio R=0 and different failure probabilities (P_f).

S	N_D					
	P_f = 0.01	P_f = 0.05	P_f = 0.10	P_f = 0.20	P_f = 0.25	P_f = 0.50
0.90	11	39	67	118	143	278
0.85	29	99	171	301	365	709
0.80	78	268	462	814	986	1914
0.75	225	772	1329	2342	2837	5510
0.70	698	2390	4115	7251	8784	17061
0.65	2351	8047	13855	24415	29576	57446
0.60	8724	29860	51416	90600	109754	213174

TABLE 5. Design fatigue life (N_D) for the stress ratio R=0.1 and different failure probabilities (P_f).

S	N_D					
	P_f = 0.01	P_f = 0.05	P_f = 0.10	P_f = 0.20	P_f = 0.25	P_f = 0.50
0.90	12	51	97	188	236	515
0.85	34	143	271	527	661	1442
0.80	100	426	807	1572	1970	4300
0.75	321	1364	2583	5030	6302	13758
0.70	1112	4727	8957	17438	21849	47698
0.65	4228	17971	34050	66292	83064	181331
0.60	17887	76033	144060	280467	351424	767172

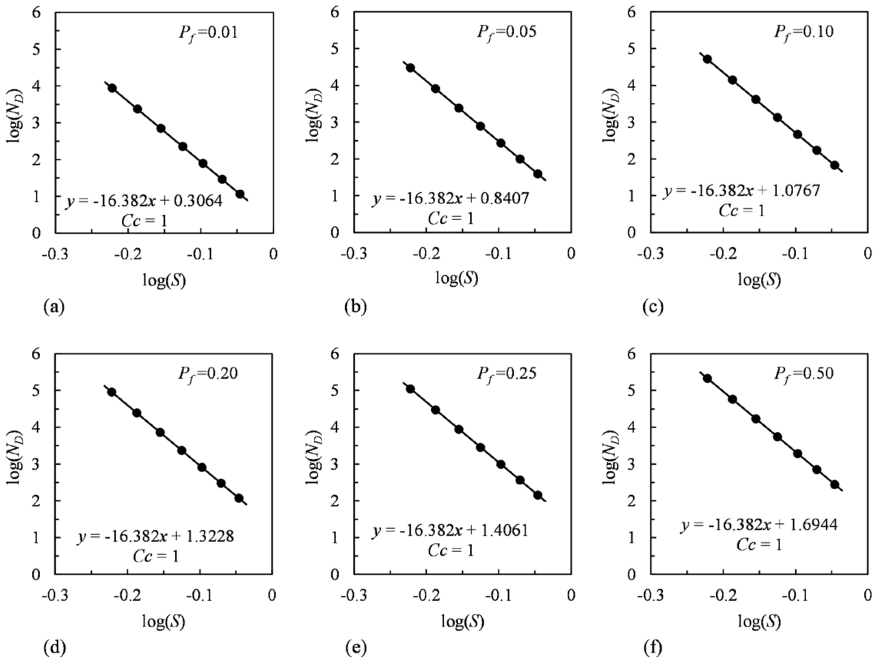

FIGURE 3. S-N-P_f curves for stress ratio R = 0 and the failure probabilities of (a) 0.01, (b) 0.05, (c) 0.10, (d) 0.20, (e) 0.25, and (f) 0.50.

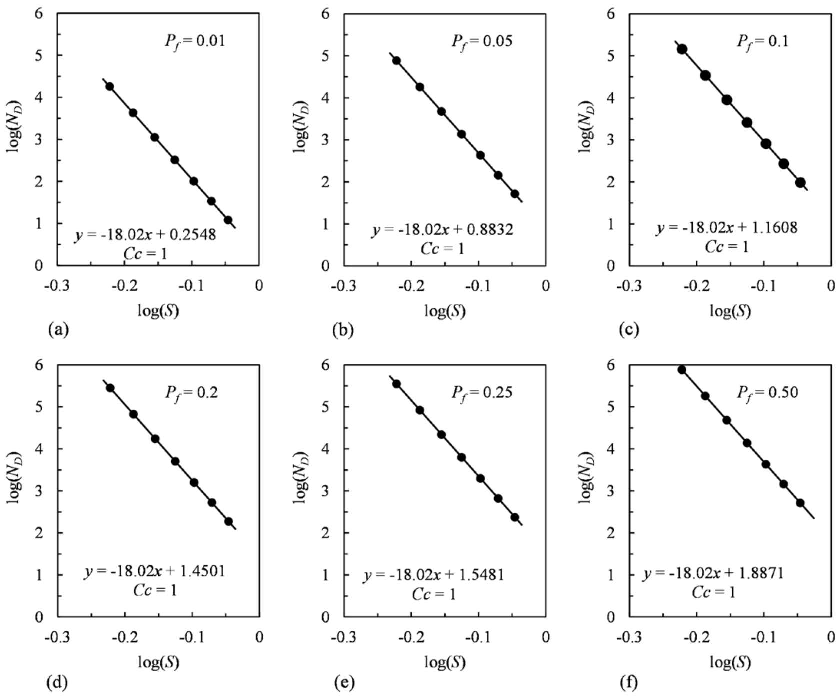

FIGURE 4. S-N-P_f curves for stress ratio R = 0.1 and the failure probabilities of (a) 0.01, (b) 0.05, (c) 0.10, (d) 0.20, (e) 0.25, and (f) 0.50.

5. DISCUSSION OF THE PROPOSED FATIGUE EQUATIONS

5.1 Wöhler fatigue equations

The Wöhler fatigue equations for NWC were generated using the equivalent fatigue-life data discussed in section 2.2. Using Equation [1] and the coefficients in Table 1, the Wöhler fatigue equations were generated for the stress ratios R = 0 and R = 0.1, as shown in Table 6 below. From these equations, the fatigue strength or the fatigue stress level (S) could be determined for the desired number of loading cycles.

The fatigue strength and the endurance limits are important design parameters for the structures (e.g., bridge decks, highways, and airfield pavements) which are subjected to repeated loads because these structures are designed based on the endurance limit of the concrete. Most of the studies (27–29) defined the endurance limit as the maximum stress level at which structures are able to withstand two million cycles of irreversible repetitive loading. Generally, this stress level is expressed as a percentage of the static flexural strength of the concrete. The fatigue strengths for one million and two million cycles were calculated using the Wöhler fatigue equation for two different stress ratios, as shown in Table 6. It can be seen in the table that the calculated endurance limit of NWC is 56% and 61% of the static flexural strength respectively for the stress ratios $R = 0$ and $R = 0.1$.

Table 6. Wöhler fatigue equations and the fatigue strengths (S) for one and two million cycles.

Stress ratio (R)	Wöhler fatigue equation	S for	
		1×10^6 cycles	2×10^6 cycles
0.0		0.58	0.56
0.1		0.63	0.61

5.2 Fatigue equation using S–N power relationship

Using Equation [4] and the empirical constant values given in Table 2, the S–N power fatigue equation or double-logarithm fatigue equation for NWC was generated. The generated fatigue equations for the stress ratios $R = 0$ and $R = 0.1$ are shown in Table 7 below. Using these double-logarithm fatigue equations, the fatigue strengths (S) for two million cycles are 0.52 and 0.56 respectively for the stress ratios $R = 0$ and $R = 0.1$. Using the Wöhler fatigue equations, however, the flexural fatigue strengths of NWC for 2×10^6 cycles were found to be 0.56 and 0.61 respectively for the stress ratios $R = 0.0$ and $R = 0.1$. Therefore, the fatigue strength for 2 million cycles using the double-logarithm fatigue equation (S–N power relationship) is more conservative than that derived by using the Wöhler fatigue equation.

Table 7. S–N power relationship or double-logarithm fatigue equation for NWC.

Stress ratio (R)	Double-logarithm fatigue equation	S for	
		1×10^6 cycles	2×10^6 cycles
0.0		0.54	0.52
0.1		0.59	0.56

The fatigue strength of NWC for one or two million cycles of loading obtained by the Wöhler and the double-logarithm fatigue equations was compared to the results published by other studies. Based on the analysis of limited test results, Ramakrishnan *et al.* (28) suggested the endurance limit (2 million cycles) to be approximately 50–55% of the static ultimate flexural strength for stress ratio $R = 0.0$. In the present study using the double-logarithm fatigue equations, the endurance limit was found to be 52% of the static ultimate flexural strength for the stress ratio $R = 0.0$. This result was found using a large quantity of test data, which is more reliable than the results Ramakrishnan *et al.* (28) obtained from a single set of test data. According to ACI 215R-74 (30), the fatigue strengths of NWC for one million cycles are 49.66% and 56% of the static ultimate flexural strength respectively for the stress ratios $R = 0.0$ and $R = 0.1$. Using the double-logarithm fatigue equation (Table 7), the fatigue strengths for one million cycles are 54% and 59% of the flexural strength respectively for the stress ratios $R = 0.0$ and $R = 0.1$ (see Table 7). This shows that the ACI recommendation is slightly conservative compared to the result obtained from the double-logarithm fatigue equation generated using a large number of fatigue-life data. Because the proposed double-logarithm fatigue equation gives a close approximation of fatigue strength in comparison to other research and code recombination, the fatigue equations given in Table 7 may be recommended for design purposes for NWC (compressive strength range of 25–60 MPa). Furthermore, it can also be concluded that the proposed double-logarithm fatigue equation (S–N power relationship) estimates a more reasonable flexural fatigue strength for one or two million load cycles than that calculated by the Wöhler fatigue equation.

5.3 Fatigue equations using failure probability

The design fatigue lives of NWC for the failure probabilities of 0.01, 0.05, 0.10, 0.20, 0.25, and 0.50 were calculated, as discussed above, using the Weibull distribution parameters (α and u). The design fatigue lives were plotted against stress levels for each failure probability (P_f). Fatigue equations were obtained by linear regression analysis of the plotted data, as shown in Figures 3 and 4. For the failure probabilities of 0.01, 0.05, 0.10, 0.20, 0.25, and 0.50 the generated fatigue equations are given in Tables 8 and 9 for stress ratios $R = 0$ and $R = 0.1$. The fatigue strength for 2×10^6 cycles is also presented in these tables. From the equations in Tables 8 and 9 it can be shown that the design fatigue strength is reduced for a given fatigue life, with a concomitant lower probability of failure.

Table 8. Fatigue equation of NWC for $R = 0.0$ and different failure probabilities.

P_f	Fatigue equation	S^*
0.01	$\log(N) = 0.3064 - 16.382 \log(S)$	0.43
0.05	$\log(N) = 0.8407 - 16.382 \log(S)$	0.46
0.10	$\log(N) = 1.0767 - 16.382 \log(S)$	0.48
0.20	$\log(N) = 1.3228 - 16.382 \log(S)$	0.50
0.25	$\log(N) = 1.4061 - 16.382 \log(S)$	0.50
0.50	$\log(N) = 1.6944 - 16.382 \log(S)$	0.52

$^*S$ for 2×10^6 cycles

Table 9. Fatigue equation of NWC for $R = 0.1$ and different failure probabilities.

P_f	Fatigue equation	S^*
0.01	$\log(N) = 0.2548 - 18.02 \log(S)$	0.46
0.05	$\log(N) = 0.8832 - 18.02 \log(S)$	0.50
0.10	$\log(N) = 1.1608 - 18.02 \log(S)$	0.52
0.20	$\log(N) = 1.4501 - 18.02 \log(S)$	0.54
0.25	$\log(N) = 1.5481 - 18.02 \log(S)$	0.55
0.50	$\log(N) = 1.8871 - 18.02 \log(S)$	0.57

$^*S$ for 2×10^6 cycles

Generally, the failure probability of 0.5 represents the mean fatigue life of the concrete (18). It is also found that the flexural fatigue strength for 2×10^6 cycles obtained by including the failure probability of 0.5 ($P_f = 0.5$) is less than that obtained using the Wöhler fatigue equation. However, the fatigue strength for $P_f = 0.5$ is similar to those obtained by the double-logarithm fatigue equation (*S–N* power relationship).

In this study, only the two-parameter Weibull distribution was used to perform the probabilistic analysis. However, it will be good to perform a comparative study using the Gumbel, Weibull and log-normal distribution in the future study. From the comparative study, the best mathematical model for fatigue-life of NWC can be recommended for design.

6. CONCLUSIONS

The flexural fatigue test data for plain NWC were collected from 16 different sources available in the literature. The concept of the equivalent fatigue life was used to remove the effect of stress ratios and arrive at the fatigue life of concrete using the same stress ratio. The *S–N* curves were generated using two different methods, and probabilistic analyses were carried out to develop S-N-P_f curves for different failure probabilities. The main concluding remarks are as follows:

1. Using equivalent fatigue lives for two different stress ratios, Wöhler and double-logarithm fatigue equations were generated. Using these fatigue equations, the flexural fatigue strength for specific cycles of loading could be determined.
2. According to the double-logarithm fatigue equation, the fatigue strength for two million cycles is 52% and 56% of the static flexural strength respectively for the stress ratios $R = 0.0$ and $R = 0.1$. In comparison, these values are 56% and 61% of the static flexural strength respectively according to the Wöhler fatigue equation for the stress ratios $R = 0.0$ and $R = 0.1$.
3. The double-logarithm fatigue equation estimates a more reasonable flexural fatigue strength for one and two million fatigue cycles than the Wöhler fatigue equation. Therefore, the fatigue equations obtained by the *S–N* power relationship may be recommended to predict the design fatigue strength of NWC (compressive strength range of 25–60 MPa).
4. Considering the fatigue strength together with failure probability leads to more conservative conclusions than that without considering the failure probability. For the safe design of the concrete structures under flexural loading, the developed fatigue equations (Tables 8 and 9) incorporating the failure probability may be used.

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AUTHOR CONTRIBUTIONS:

Conceptualization: K. M. A. Sohel, M. H. S. Al-Hinai, A. Alnuaimi, M. Al-Shahri, S. El-Gamal. Data curation: K. M. A. Sohel, M. H. S. Al-Hinai. Formal analysis: K. M. A. Sohel; M. H. S. Al-Hinai. Funding acquisition: K. M. A. Sohel. Investigation: K. M. A. Sohel, M. H. S. Al-Hinai. Methodology: K. M. A. Sohel, M. H. S. Al-Hinai, A. Alnuaimi, M. Al-Shahri, S. El-Gamal. Project administration: K. M. A. Sohel. Resources: K. M. A. Sohel, A. Alnuaimi. Software: K. M. A. Sohel, M. H. S. Al-Hinai. Supervision: K. M. A. Sohel, A. Alnuaimi; M. Al-Shahri, S. El-Gamal. Validation: K. M. A. Sohel, M. H. S. Al-Hinai. Visualization: K. M. A. Sohel, M. H. S. Al-Hinai, A. Alnuaimi, M. Al-Shahri, S. El-Gamal. Writing, original draft: K. M. A. Sohel, M. H. S. Al-Hinai. Writing, review & editing: K. M. A. Sohel; M. H. S. Al-Hinai; A. Alnuaimi; M. Al-Shahri; S. El-Gamal.

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Appendix A: Flexural fatigue-life data of normal weight concrete

TABLE A1. Fatigue test results of plain normal weight concrete obtained from different experimental studies.

f_c, f_r, and R	Fatigue life N for different stress level S							
	0.95	0.90	0.85	0.8	0.75	0.7	0.65	0.60
Oh, [20]			1038		15210		164097	
f_c = 27 MPa;			1064		15618		176071	
f_r = 4.58 MPa;			1620		19286		233916	
R = 0.1			1758		19598		245794	
			1770		19849		249906	
			1814		20694		256619	
			1872		21046		293559	
			1940		21334		334895	
			1954		23662		358636	
			2047		24345		385780	
			2107		24820		435673	
			2162		40809		635258	
			2620		52516		724621	
			3150		--		--	
Arora and Singh [27]			444*		10781		100801	
f_c = 41.77 MPa;			1137		13879		142054	
f_r = 5.10 MPa;			1367		18489		187623	
R =0.1			1678		21945		220075	
			1945		25467		260685	
			2271		31256		323068	
			2605		36543		360845	
			2647		42842		456944	
			3096		46951		512089	
			3987		51348		558973	
Kaur *et al.* [24]		682	991**	2607**	137850			
f_c = 40.18 MPa;		754	7778	34205	238539			
f_r = 8.08 MPa;		914	9380	46786	278282			
R =0.10		982	15399	71264	418536			
		1387	17478	93839	568538			
		1584	30053	104357	584786			
		1835	38659	131770	625875			
		2144	40327	203351	1692796			
		2544	61244	279493	1846843			
		2865	74546	415850	2000000*			

TABLE A1 (cont.). Fatigue test results of plain normal weight concrete obtained from different experimental studies.

f_c, f_r, and R	Fatigue life N for different stress level S							
	0.95	0.90	0.85	0.8	0.75	0.7	0.65	0.60
Mohammadi and Kaushik [14] f_c = 58 MPa; f_r = 5.35 MPa; R =0.1			942	4664		53322		
			1205	5655		56453		
			1290	5963		59493		
			1347	6614		63997		
			1386	6773		68387		
			1593	7621		81038		
			1664	8903		94102		
			1781	9379		114214		
			1902	10986		138563		
			2644	15385		189550		
			4482*	-		288054*		
Liu *et al.* [31] f_c = 43.4 MPa; f_r = 5.6 MPa; R =0.1		450		12215		27945		39480
		535		14941		28935		48245
		1090		20686		36365		59910
Harwalkar and Awanti [32] f_c = 62.30 MPa; f_r = 6.90 MPa; R =0.1			22	84	158	1327	5289	16488
			43	97	284	1489	7213	20312
			69	105	312	2596	8863	22268
			78	152	382	3642	10322	34511
			82	184	411	4149	12723	39920
			94	198	474	5218	16523	46718
			102	288	578	6629	18708	51512
			110	432	694	8383	20391	61512
			122	682	916	9558	21262	77812
			138	730	1182	12009	23992	81800
			-	-	-	-	24771	92477
			-	-	-	-	27344	100000*
			-	-	-	-	32811	100000*
			-	-	-	-	40887	100000*
			-	-	-	-	44816	100000*
Johnston and Zemp [29] f_c = 51.00 MPa; f_r = 4.45 MPa; R =0.1		10	100	5100	5000			
		45	400	8000	35000			
		55	1610	11300	71000			
		195	2800	16200	100000			
		260	4350	32000	127000			
		365	6020	33000				
		900						
Tan et al. [33] Min^m f_c = 30 MPa; f_r = 5.9 MPa; R =0.1		239				6594		76164
		348				28156		94618
		513				41645		138495

TABLE A1 (cont.). Fatigue test results of plain normal weight concrete obtained from different experimental studies.

f_c, f_r, and R	Fatigue life N for different stress level S							
	0.95	0.90	0.85	0.8	0.75	0.7	0.65	0.60
Shi et al. [8]		9	49			10184	46836	206479
f_c = 30 MPa;			73			11808	125934	346047
f_r = 6.08 MPa;			76			21684	129009	436123
R =0.08			150			21747	150331	628962
			160			43683	159033	694263
			230			49392	164795	883301
			402			50997	166287	1956530
						70937	170426	2032902
						72266	191828	2672740
						77122	202916	
						79778		
						82905		
						100411		
						101918		
Shi et al. [8]		77		1398	22511	34206	375170	
f_c = 30 MPa;				4829		41730	566140	
f_r = 6.08 MPa;				6400		177807	618936	
R =0.2				9059		-	2011017	
				-		-	2434133	
Zhang et al. [34]	14	69	277	2330	20550	82890		
f_c = 50.70 MPa;	16	91	410	2640	22030	99220		
f_r = 7.19 MPa;	20	95	431	3310	28110	137150		
R =0.2	24	103	693	4170	31200	168100		
	26	111	744	5010	34250	208750		
	27	146	987	7460	54630	219710		
	27	163	1052	10050	62430	387100		
	28	204	1390	13230	62610	409610		
	41	462	1948	16980	152490	467990		
	76	600	2192	24330	152740	1407700		
Lee et al. [35]		40		400		30980	1053500	
f_c = 37 MPa;		80		590		82340	2109950	
f_r = 5.95 MPa;		100		1430		97230	-	
R =0.02 to 0.03		1150		6110		319700	-	
		1844		6337		915240	-	
		3140		29200		-	-	
Thomas [36]		1340		7240		33190		257570
f_c = 35.9 MPa;		1870		8400		51180		1287300
f_r = 5.79 MPa;		3240		12130		62050		2316280*
R =0.02		3570		12710		65170		2138260*
		4210		22660		127490		2112750*

TABLE A1 (cont.). Fatigue test results of plain normal weight concrete obtained from different experimental studies.

f_c, f_r, and *R*	Fatigue life N for different stress level S							
	0.95	0.90	0.85	0.8	0.75	0.7	0.65	0.60
Thomas [36]		42		401		34520	11920	
f_c = 50.90 MPa;		233		3740		47970	114440	
f_r = 5.40 MPa;		434		4300		48440		
R =0.01 to 0.02		639		9440		95190		
		953		11045		830880		
		1040		80091		-		
Hanumantharay-agouda and Patil [37]			6784	7325	19340	25349	66120	
f_c = 52.00 MPa;			8450	8735	21758	48323	69214	
f_r = 4.62 MPa; R =0.01			9042	9745	22378	49892	55397	
Paluri et al. [2]		63		2619		19983		
f_c = 52.00 MPa;		93		3553		24418		
f_r = 4.62 MPa; R =0.01		139		4478		28299		
Zhang et al. [34]		39	121	637	2830	13150	72880	
f_c = 50.70 MPa;		45	168	655	4280	18320	77800	
f_r = 7.19 MPa;		46	175	923	4530	66360	86360	
R =0		72	364	1327	-	-	-	
		94	-	-	-	-	-	
Mithun et al. [38]			53	460	7657	18714		
			71	779	16403	28623		
f_c = 57 MPa;			131	889	25458	40883		
f_r = 7.05 MPa;			278	1413	36186	63451		
R = 0			355	1636	43993	79823		