Simplified modeling of rubberized concrete properties using multivariable regression analysis

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ABSTRACT: The studies on rubberized concrete have increased dramatically over the last few years due to being an environmentally friendly material with enhanced vibration behavior and energy dissipation capabilities. Nevertheless, multiple resources in the literature have reported reductions in its mechanical properties directly proportional to the rubber content. Over the last few years, various mathematical models have been proposed to estimate rubberized concrete properties using artificial intelligence, machine learning, and fuzzy logic-based methods. However, these models are relatively complicated and require higher computation efforts than multivariable regression ones when it comes to the daily usage of practicing engineers. Additionally, most of the study has mainly focused on the compressive strength of rubberized concrete and rarely went into more details considering other properties and sample sizes. Therefore, this study focuses on developing simple yet accurate rubberized concrete multivariable regression models that can be generalized for various mixtures of rubberized concrete considering different sample sizes

KEY WORDS: Rubberized concrete; Structural material; Mechanical properties; Numerical correlations.

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RESUMEN: Modelización simplificada de las propiedades del hormigón con caucho empleando un análisis de regresión multivariable. Los estudios sobre hormigón incorporando caucho han aumentado drásticamente en los últimos años debido a que es un material ecológico con un comportamiento de vibración mejorado y capacidades de disipación de energía. Sin embargo, múltiples trabajos en la literatura han indicado reducciones en sus propiedades mecánicas directamente proporcionales al contenido de caucho. En los últimos años se han propuesto varios modelos matemáticos para estimar las propiedades del hormigón con caucho utilizando inteligencia artificial, aprendizaje automático y métodos basados en lógica difusa. Sin embargo, estos modelos son relativamente complicados y requieren mayores esfuerzos de cálculo que los de regresión multivariable en el día a día de los ingenieros. Además, la mayor parte de los estudios se han centrado principalmente en la resistencia a la compresión del hormigón con caucho y rara vez entran en más detalles considerando otras propiedades y tamaños de muestra. Por lo tanto, este estudio se centra en el desarrollo de modelos de regresión multivariable de hormigón con caucho, simples pero precisos, que se pueden generalizar para varias mezclas de hormigón de este tipo, considerando diferentes tamaños de muestra.

PALABRAS CLAVE: Hormigón con caucho; Material estructural; Propiedades mecánicas; Correlaciones numéricas.

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LIST OF NOMENCLATURE

| Symbol | Unit | Description |
|----------------|-------------------|--|
| RBC | - | Rubberized concrete |
| RMSE | - | root-mean-square error |
| MAPE | - | mean absolute percentage error |
| R _m | - | Rubber replacement method |
| R _c | (kg) | Rubber content |
| R _p | (%) | Rubber replacement percentage |
| ρ | kg/m ³ | The hardened density of concrete |
| ρ_c | kg/m ³ | The hardened density of control con- crete |
| ρ _r | kg/m ³ | The hardened density of RBC |
| f_{c28} | (MPa) | Compressive strength of concrete at 28 days |
| f_{cc} | (MPa) | Compressive strength of control con- crete at 28 days |
| f_{cr} | (MPa) | Compressive strength of RBC at 28 days |
| f_r | (MPa) | Flexural strength of concrete |
| Es | (GPa) | Static modulus of elasticity |
| E _d | (GPa) | Dynamic modulus of elasticity |
| | | |

1. INTRODUCTION

Nowadays, the disposal of old rubber tires into landfills represents a severe environmental hazard worldwide (1-3). Over the last few decades, many researchers in the construction industry have discussed recycling this waste as aggregates in the mixture of cement-based materials such as concrete (4-7). Currently, it is understood that adding rubber particles into concrete mixes changes its mechanical and dynamic properties significantly (8, 9). For instance, it was found that replacing 25% of the natural aggregates in high strength concrete mixture with well-graded rubber particles reduces its compressive strength by 43% while increasing its damping ratio by over 91% (10). This reduction in the compressive strength of rubberized concrete (RBC) was previously attributed to the strength of natural aggregate as compared to that of the rubber one (8) and the weak bond developed between the recycled aggregates and the cement matrix (11). Additionally, it was concluded that the drop in the strength properties of RBC is higher in the case of using coarse aggregates as compared to that of the fine ones (12-14). Previously, Topçu, & Sarıdemir (15) used feed-forward back-propagation neural network and an adaptive neuro-fuzzy inference system to estimate the fresh density and flow table value of RBC. On the other hand, various models were developed to predict the compressive strength of concrete, including multivariable linear and nonlinear regression (16), artificial neural network (17), adaptive neuro-fuzzy inference system (18), genetic programming (19), support vector machine (20), nonlinear formulation approach using feed-forward back-propagation neural network (21), k-nearest neighbor and random forests (22). In addition, Cheng & Cao (23) adopted evolutionary multivariate adaptive regression splines, multivariate adaptive regression splines, feed-forward back-propagation neural network, radial basis function neural network, and genetic programming to predict the compressive and splitting tensile strength of RBC. Whereas Habib & Yildirim (24) developed multivariable linear regression and feed-forward back-propagation neural network to estimate the dynamic modulus of elasticity, damping ratio, and natural frequency of RBC elements. However, most of these models are considered fairly complicated and demand high computation efforts when it comes to the daily usage of practicing engineers. Moreover, most of the study has mainly focused on the compressive strength of rubberized concrete and rarely went into more details considering other properties and sample sizes. Thus, this research proposes simplified prediction models that can be used in RBC mixtures of both normal and high strength capacity at a wide range of rubber contents for different specimen sizes. In order to do so, a large database composed of over 1000 collected experimental results for the hardened density, compressive and flexural strengths, static and dynamic moduli will be used in investigations. In general, multivariable regression models that are capable of estimating the hardened density, compressive and flexural strengths, and static modulus of elasticity will be developed. In addition, various non-destructive testing correlations between the mechanical and dynamic modulus of elasticity will be proposed.

2. MATERIALS AND METHODS

2.1. Collection of rubberized concrete properties

The final dataset, Table 1, was based on about 1000 experimental findings from 28 papers that discussed the performance of rubberized concrete. Accordingly, the content of natural fine and coarse aggregates, rubber replacement method, rubber content, and rubber replacement ratio were collected from each paper in addition to the reported mechanical and dynamic properties of the produced concrete. In general, the concrete mixtures that were considered in this study represented conventional concrete or concrete with silica fume and/or fly ash, while those with fibers were ignored. Moreover, during the database generation stage, issues such as the size of the test specimen, concrete mixture's age at testing, and repetition of the same findings in other papers/reports were considered carefully to prevent any problem during the analysis.

TABLE 1. Collected datasets for the numerical study.

| A | 1 | Rubber | r | Concrete Properties | | | | |
|----------|----------------|----------------|----------------|---------------------|-------------------------|--------------|--------------|----------------|
| Authof/s | R _m | R _c | R _p | ρ | <i>f</i> _{c28} | f_r | Es | E _d |
| (8) | ~ | - | ~ | ~ | ~ | - | 1 | - |
| (9) | - | \checkmark | - | 1 | ~ | - | ~ | \checkmark |
| (14) | ~ | - | \checkmark | - | ~ | - | - | - |
| (25) | 1 | ~ | ~ | ~ | ~ | - | 1 | \checkmark |
| (26) | ~ | \checkmark | \checkmark | - | ~ | - | - | \checkmark |
| (27) | 1 | ~ | ~ | ~ | - | - | 1 | \checkmark |
| (28) | 1 | ~ | ~ | ~ | ~ | - | 1 | \checkmark |
| (29) | 1 | ~ | ~ | ~ | ~ | 1 | 1 | \checkmark |
| (30) | ~ | \checkmark | \checkmark | \checkmark | ~ | \checkmark | - | - |
| (31) | 1 | \checkmark | \checkmark | - | ~ | - | 1 | \checkmark |
| (32) | 1 | ~ | ~ | ~ | ~ | 1 | - | \checkmark |
| (33) | 1 | ~ | ~ | - | - | - | - | \checkmark |
| (34) | ~ | - | \checkmark | 1 | \checkmark | - | ~ | - |
| (35) | ~ | \checkmark | \checkmark | 1 | - | - | - | - |
| (36) | ~ | \checkmark | \checkmark | - | - | \checkmark | ~ | - |
| (37) | ~ | - | \checkmark | 1 | - | - | - | - |
| (38) | ~ | \checkmark | \checkmark | 1 | - | \checkmark | ~ | - |
| (39) | ~ | \checkmark | \checkmark | 1 | \checkmark | \checkmark | ~ | - |
| (40) | ~ | \checkmark | \checkmark | - | - | \checkmark | - | - |
| (41) | ~ | \checkmark | \checkmark | 1 | - | \checkmark | - | - |
| (42) | 1 | \checkmark | \checkmark | 1 | ~ | - | 1 | - |
| (43) | 1 | \checkmark | \checkmark | - | - | 1 | - | - |
| (44) | ~ | \checkmark | \checkmark | - | ~ | - | - | - |
| (45) | ~ | \checkmark | \checkmark | 1 | - | \checkmark | - | - |
| (46) | ~ | \checkmark | - | \checkmark | ~ | - | \checkmark | - |
| (47) | ~ | \checkmark | \checkmark | - | ~ | - | \checkmark | - |
| (48) | \checkmark | - | \checkmark | - | - | \checkmark | - | - |

2.2. Prediction strategy

indeed, several obstacles can be encountered when it comes to proposing a generalized regression model for RBC mixtures with fairly controlled errors due to the variation in the type of binding materials being used, the concrete mixture compositions, and inconsistency in using a specific rubber replacement method. In fact, the effects of the cement type and the concrete mixture composition can be directly seen on the control specimen's properties when comparing two types of concrete together. Thus, to take the influence of these factors into account, this study uses a parameter that can reflect the variation of the mix design on the properties of the concrete as an input to the regression model.

Accordingly, the main aim of using the property of the control concrete as input to the regression model is to account for the influence of the concrete mixture properties (i.e., water/cement ratio, cement type, the addition of chemical and mineral admixtures, density of natural aggregates as well as its particle size distribution). This technique is expected to improve the outcomes of the prediction and allow the generalization of the developed models. In this regard, the models would be used to design rubberized concrete mixture and optimize the rubber content into a given concrete mix proportion. On the other hand, to overcome the problem of the inconsistency in the rubber replacement method where some studies replaced the natural stone particles by volume while others did it by weight, this study uses the rubber content represented by the weight of the rubber in kg the mixture as input to the proposed models. Moreover, this study does not consider the influence of the rubber particle size distribution when developing the regression equations. The main aim behind this comes from the inconsistency of the literature in that matter. For instance, some papers have used single-graded rubber particles while other utilized well-graded ones. Additionally, some used a range of 2 mm to 4 mm, while others had 6 mm to 12 mm in a single concrete mixture. This issue has caused a huge difficulty in numerically incorporating the influence of the rubber particle size on strength reduction. On the other hand, to overcome this issue and prevent any limitation in the model to specific rubber particle size distribution, it is suggested to use data for fine and coarse rubber aggregates in which the proposed models can be generalized for a wide range of cases at the cost of for slightly higher distortion in the results.

2.3. Empirical formulation and regression analysis

in general, multivariable linear regression analysis is a statistical method used to model the relationship between multiple independent variables an output response. The mathematical model used in this approach is given in Equation [1] (49):

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon$$
[1]

where y represent the list observations on the dependent variable; x_1, \ldots, x_k are the independent variables; β_0 is an intercept term; β_1, \ldots, β_k are the coefficients to be estimated; and ε is a list of random errors, also known as the residual.

On the other hand, the performance of the multivariable linear regression can result in an underfitting of the data to its nature of simplicity. For such purposes, engineers and scientists tend to use nonlinear empirical formulations that contain multiple coefficients in line with the input variables to reach a high predictability level. These equations are not necessarily systematic such as the multivariable linear regression ones but rather are prepared on a case-by-case routine to fit the given dataset. Indeed, various types of empirical formulation are available in the literature where they include polynomial functions while others were exponential and even logarithmic.

To optimize the constants in the function, multiple methods were adopted in the literature, such as the least-squares method (50), nonlinear least-squares procedure (51), and sequential least squares programming algorithm (52, 53).

In this study, the nonlinear least-squares procedure was adopted to achieve the optimal solution for each of the developed models.

Let's consider an arbitrary empirical equation given in Equation [2]:

$$f(x) = a + e^{\frac{b}{x_1}} + cx_2^2$$
 [2]

where a, b, and c are the parameters to be fitted and x_1 and x_2 are the input variables.

In a matrix form, this equation can be written as follows:

$$y = f(X,\beta) + \varepsilon$$
 [3]

where y is the list of observations on the dependent variable; X are the independent variables; β is an intercept term to represent the list of coefficients to be optimized, and ε is a list of residuals.

The nonlinear least-squares procedure tries to solve the objective function in Equation [4] to minimize the sum squared residuals.

$$\min(\|y_i - F(X_i)\|_2^2)$$
 [4]

Once the empirical models are developed, it is important to estimate errors. This is usually done by performing a residual analysis. The residuals of an i^{th} observation is defined in Equation [5]:

$$e_i = (y_i - \hat{y}_i)$$
[5]

Additionally, the goodness-of-fit or proportion of the variance of each developed model can be represented by the coefficient of determination (Equation [6]):

$$R^{2} = 1 - \frac{\sum (y_{i} - \hat{y}_{i})^{2}}{\sum (y_{i} - \overline{y})^{2}}$$
[6]

where y_i is the actual value, \hat{y}_i is the predicted one, and \overline{y} is the mean of the actual values.

2.4. Error analysis

in fact, measuring the accuracy of a particular prediction modeling is not an easy task. Currently, several methods for calculating the error are available. In this study, root-mean-square error (RMSE), Equation [7], and mean absolute percentage error (MAPE), Equation [8], will be used to evaluate the estimation models.

RMSE =
$$\sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}}$$
 [7]
MAPE = $\frac{100}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right|$ [8]

where y_i is the actual value, \hat{y}_i is the predicted one, and *n* is the number of observations.

3. RESULTS AND DISCUSSIONS

3.1. Prediction of hardened density

the descriptive statistics of the data used for proposing the hardened density model are shown in Table 2. The constructed mathematical expression in this section is generally shown in Equation [9], and its performance is investigated in Figure 1. Generally, the R^2 value of this model is equal to 0.898, which gives a good sense of accuracy in prediction.

TABLE 2. Descriptive statistics for the dataset used in the hardened density model.

| Statistics | R _c | $ ho_{c}$ | ρ_r |
|--------------------|----------------|-----------|----------|
| Sample size | 102 | 102 | 102 |
| Mean | 111.8 | 2412.4 | 2215.2 |
| Standard error | 12.9 | 11.1 | 31.1 |
| Standard deviation | 129.9 | 112.5 | 314.1 |
| Minimum | 7 | 2213.7 | 1086.9 |
| First quartile | 36.2 | 2274.8 | 2161.5 |
| Median | 63.5 | 2421.3 | 2324.8 |
| Third quartile | 129.7 | 2508.9 | 2407.7 |
| Maximum | 609.5 | 2546.7 | 2524 |

This is also indicated in the prediction model's performance against the measured values, Figure 1-a, and the difference between the measured values and the predicted ones (residual), Figure 1-b.

$$\rho_r = \rho_c - 2.7 R_c^{0.92}$$
 [9]

In fact, due to the significant difference in the rubber aggregates' weight compared to the natural ones, RBC can have the density of normal concrete or lightweight ones based on the amount of natural aggregates being replaced. Thus, the capability of this model in estimating both normal and lightweight RBC mixtures can be seen in Figure 1-c. It can be noticed that the model provided very high accuracy in predicting the density of RBC up to 2200 kg/m^3 , whereas a lower accuracy is faced when the density goes below that reaching a residual value of almost 150 kg/m^3 . This point can also be observed from the box plots provided in Figure 1-c, in which the high side of the predicted box has better matching to the measured values compared to the lower one. This reduced capabilities in the model when it comes to lightweight concrete can be attributed to the lesser data available on RBC density with a considerably high amount of rubber that can result in a significant density reduction.

The analysis of the error in this model can be seen in Table 3 in which the standard deviation of the residuals is almost 100 kg/m³ and the MAPE was 3.37%. Thus, this model can be reliably used for predicting the hardened density of RBC.

| Error Type | Model Performance |
|------------|-------------------|
| RMSE | 99.67 |
| MAPE (%) | 3.37 |

TABLE 3. Error analysis of the hardened density prediction model.

3.2. Prediction of compressive strength

in this section, a simplified model for predicting the compressive strength of concrete based on a very wide range of rubber content and different types of rubber replacement methods, including replacement by fine, coarse, or total natural aggregates. Moreover, the model can be used to predict the compressive strength of concrete with different specimen sizes. The sample size of the dataset used was 174 RBC results, with the descriptive statistics shown in Table 4.

The mathematical expression of the estimation model is given in Equation [10], and the R² value of this model was 0.766, whereas its fitting performance was analyzed in detail, as shown in Figure 2:



FIGURE 1. Performance of the RBC hardened density prediction model.

| Statistics | R | f_{cc} | f_{cr} |
|-------------------------|-------|----------|----------|
| Sample size | 174 | 174 | 174 |
| Mean | 82.35 | 54.14 | 33.3 |
| Standard error | 6.43 | 1.23 | 1.04 |
| Standard devi- ation | 84.86 | 16.28 | 13.72 |
| Minimum | 6.95 | 27.62 | 6.94 |
| First quartile | 35.04 | 39 | 22.51 |
| Median | 55.4 | 56.1 | 31.5 |
| Third quartile | 80.57 | 63.2 | 42.38 |
| Maximum | 490.3 | 104.8 | 70.5 |

 $f_{cr} = f_{cc} - 0.155R_c + 0.0005R_c^2 - 0.003f_{cc}R_c$ [10]

In general, it can be seen that the proposed regression model provides reasonably high perfor-

TABLE 4. Descriptive statistics for the dataset used in the compressive strength model.

mance in predicting the compressive strength of rubberized concrete with different specimen sizes, as presented in Figure 2-a and Figure 2-c. On the other hand, the residuals of the estimated values, Figure 2-b, show that the error in the model's prediction is not dependent on the specimen type or size. Furthermore, the box plots of the measured and predicted values, Figure 2-c, depict that this model could estimate the general trends and statistical properties of the dataset.

The error analysis in this section is shown in Table 5. It can be seen that the MAPE value was about 20.28% which represents that this model can be reliably used for predicting the compressive strength of RBC mixtures.

TABLE 5. Error analysis of the compressive strength prediction model.

| Error Type | Model Performance | |
|------------|-------------------|--|
| RMSE | 6.63 | |
| MAPE | 20.28 | |



(c)

FIGURE 2. Performance of the RBC compressive strength prediction model.

3.3. Prediction of static modulus of elasticity

the static modulus of elasticity is indeed a critical parameter for the analysis and design of reinforced concrete structures. Currently, several relationships for estimating this parameter based on the 28 days compressive strength of concrete are available in design codes. In this section, the capability of ACI 318 (54), Equation [11], and ACI 363 (55), Equation [12], methods in the case of RBC will be investigated, and enhanced ones will be proposed. The aim behind using different ACI models is that the collected dataset ranges between normal and high strength concrete, which means there might be a lack of predictability in one of the models. Unlike previous models with multiple inputs, the proposed equation in this section for the modulus of elasticity of RBC is developed in a similar form to the ACI standard to provide an alternative fit-for-purpose method as given in the US codes practice but with better performance for RBC mixtures. Moreover, the descriptive statistics of the datasets used in this analysis are shown in Table 6.

The collected static modulus of elasticity findings in this study were all from tests on either 100 mm and 150 mm cylinders so that these specimens follow ASTM C192 (56) requirements. Generally, ACI 318 (54) and ACI 363 (55) explicitly permit the compressive strength test to be performed on cylinders of either 100 mm diameter or 200 mm diameter. Accordingly, these two sizes were defined as one category in developing the database of this section so that the prediction models in the design codes can be applied correctly. Therefore, three different models for predicting the static models of elasticity of concrete based on the size of the compressive strength test were used herein, which are the 100 mm cubic specimen, 150 mm cubic one, and cylinders both of 100 mm and 150 mm diameters where the later one was compared to the code-based estimation equations.

ACI 318 (54)
$$E_s = 4.7\sqrt{f_{c28}}$$
 [11]

ACI 363 (55)
$$E_s = 3.32\sqrt{f_{c28}} + 6.9$$
 [12]

The prediction models are given in Equation [13], and their performances are investigated in Figure 3. In general, similar estimation capabilities are expected when using any of the proposed models since their R² values are close to each other, especially those with cubic specimens. A comparison between the measured and predicted values could be seen in Figure 3-b and Figure 3-c. Furthermore, the residuals plot is provided in Figure 3-d. It can be seen that the prediction models provide acceptable accuracy. Moreover, the regression line in Figure 3-b and the box plot, Figure 3-e, compares the capability of the prediction models in terms of the entire dataset with mixed sizes. It can be seen that the models have achieved an excellent matching to the reference dataset and were able to estimate its behavior reliably.

$$E_s = \alpha \sqrt{f_{c28}}$$
[13]

where E_s is the modulus of elasticity of concrete, f_{c28} is the 28 days compressive strength of concrete, and α is a coefficient related to the size of the compressive strength specimen as shown in Table 7.

TABLE 7. Values of α for each specimen size.

| Specimen Size | α |
|---------------|-------|
| 100 Cube | 3.889 |
| 150 Cube | 4.245 |
| Cylinders | 5.075 |

A comparison between the code-based prediction approaches and the proposed one for cylinder specimens can be seen in Figure 4. Generally, it can be

| Statistics | 100 Cube | | 150 |) Cube | Cylinders | |
|--------------------|----------|--------|-------|--------|-----------|--------|
| Statistics | Input | Output | Input | Output | Input | Output |
| Sample size | 55 | 55 | 31 | 31 | 55 | 55 |
| Mean | 38.33 | 23.831 | 37.58 | 24.87 | 30.68 | 26.89 |
| Standard error | 1.88 | 0.534 | 3.36 | 1.62 | 2.19 | 1.21 |
| Standard deviation | 13.91 | 3.958 | 18.68 | 9.02 | 16.21 | 8.97 |
| Minimum | 16.4 | 14.5 | 15.32 | 11.15 | 6.94 | 10.58 |
| First quartile | 28 | 20.9 | 22.49 | 18.9 | 20.6 | 21.14 |
| Median | 35.85 | 23.6 | 34.95 | 24.32 | 29.7 | 25.37 |
| Third quartile | 47 | 26.82 | 48.58 | 28.67 | 38.11 | 32.68 |
| Maximum | 75.3 | 32.4 | 104.8 | 56.86 | 98.25 | 56.86 |

TABLE 6. Descriptive statistics for the dataset used in the static modulus of elasticity models.



FIGURE 3. Performance of the static modulus of elasticity proposed model.

observed that the ACI 363 (55) method provided the least capability in predicting the static modulus of elasticity of the concrete mixtures. On the other hand, both the proposed model and the ACI 318 (54) give a similar behavior, with the proposed one having slightly better performance and lower residual values. This can be further highlighted from scatter plots in Figure 4-e the box plots, Figure 4-f, where the proposed approach provided the best matching as compared to the measured values.

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FIGURE 4. Applicability of code-based static modulus of elasticity estimation models for RBC.

The error analysis of the investigated models is given in Table 8. On the other hand, the proposed model provided the lowest errors as compared to ACI 318 and ACI 363 except for the MAPE, where the ACI 318 had a slightly lower value. This can be attributed to the fact that the MAPE measures the average error which means that even if most of the predicted values have small residuals, an error in a certain measurement would influence the MAPE value significantly due to the lack of weight-based computation in this method.

TABLE 8. Error analysis of the static modulus of the elasticity prediction model.

| Specimen Size | Error Type | Proposed Model | ACI 318 | ACI 363 |
|------------------|------------|-------------------|---------|---------|
| 100 Cuba | RMSE | 1.91 | - | - |
| 100 Cube | MAPE | 6.72 | - | - |
| 150 Cube | RMSE | 4.49 | - | - |
| | MAPE | 14.27 | - | - |
| Cylinders | RMSE | 3.44 | 4.02 | 5.32 |
| | MAPE | 10.68 | 10.28 | 13.92 |

3.4. Prediction of flexural strength

A prediction model for the flexural capacity of RBC based on its compressive strength is inves-

tigated in this section. In similar to the previous section, three mathematical models were developed based on the specimen size of the compressive strength test. The descriptive statistics of the utilized datasets for fitting the models can be seen in Table 9.

 TABLE 9. Descriptive statistics for the dataset used in the flexural strength models.

| G | 100 Cube | | 150 Cube | | | Cylinders | | | |
|--------------------|----------------|--------------|----------|----------------|--------------|-----------|----------------|--------------|-------|
| Statistics | R _c | $f_{_{c28}}$ | f_r | R _c | $f_{_{c28}}$ | f_r | R _c | $f_{_{c28}}$ | f_r |
| Sample size | 52 | 52 | 52 | 21 | 21 | 21 | 15 | 15 | 15 |
| Mean | 38.8 | 37.5 | 5.48 | 38.9 | 37.7 | 3.49 | 175 | 28 | 2.72 |
| Standard error | 4.48 | 2.09 | 0.17 | 6.21 | 2.9 | 0.14 | 45.8 | 3.56 | 0.27 |
| Standard deviation | 32.3 | 15.1 | 1.21 | 28.5 | 13.3 | 0.62 | 177 | 13.8 | 1.04 |
| Minimum | 0 | 17 | 3.7 | 0 | 15.3 | 2.59 | 0 | 6.94 | 1.06 |
| First quartile | 14.7 | 25 | 4.49 | 19.9 | 26.4 | 2.89 | 39.5 | 19 | 1.76 |
| Median | 36.8 | 33.6 | 5.24 | 39.8 | 37.5 | 3.46 | 123 | 27.9 | 2.57 |
| Third quartile | 50.9 | 46.6 | 6.22 | 59.7 | 49.5 | 4.06 | 362 | 36.2 | 3.47 |
| Maximum | 132 | 71 | 8.42 | 79.6 | 62.2 | 4.39 | 490 | 61 | 4.5 |

In fact, ACI 318 design code provides an equation for estimating the flexural strength of concrete. This equation requires defining a coefficient to take the density of normal weight concrete or the equilibrium density of the lightweight one into account. In section 3.1, it was shown that RBC density can vary significantly based on the utilized rubber content and in some cases, RBC mixtures reach as low as almost 1100 kg/m³ even though the control concrete has a normal weight. Thus, to take the influence of rubber aggregates on the unit weight of concrete into consideration, this study suggests using the rubber content as an input in the prediction equation. The proposed models are given in Equation [14] to Equation [16]. The R² values were calculated as 0.712, 0.947, and 0.817 for the 100 cube, 150 cube, and cylinders equations. Moreover, the performances of these estimation formulas are investigated in Figure 5.

100 Cube
$$f_r = f_{c28}^{0.5} + \frac{14R_c}{10^3} - 1.07$$

150 Cube
$$f_r = f_{c28}^{0.4} + \frac{5.8R_c}{10^3} - 0.95$$
 [15]

Cylinders
$$f_r = f_{c28}^{0.35} - \frac{2.3R_c}{10^3}$$
 [16]

It can be seen from Figure 5-a, Figure 5-b, and Figure 5-c that the proposed models provide high performance with generally small residual values regardless of the size of the specimen being tested in compression.

Furthermore, the error analysis presented in Table 10 indicates that the models can be used reliably to estimate the flexural strength of RBC concrete.

3.5. Relation between compressive strength and dynamic modulus of elasticity

In fact, concrete testing approaches are be divided into destructive and non-destructive ones. Gener-



[14]

FIGURE 5. Performance of the flexural strength prediction model.

| Specimen Size | Error Type | Model Performance |
|---------------|------------|-------------------|
| 100 Cuba | RMSE | 0.64 |
| 100 Cube | MAPE | 7.56 |
| 150 Cube | RMSE | 0.14 |
| | MAPE | 3.02 |
| Culindona | RMSE | 0.43 |
| Cylinders | MAPE | 15.99 |

TABLE 10. Error analysis of the flexural strength prediction model.

ally, destructive tests provide more reliable results as compared to non-destructive ones. However, the required procedure is rather long and expensive and will slightly or entirely damage the concrete member (57). Thus, in certain cases of practical applications, engineers tend to utilize nondestructive methods for fast measurements. Previously, several researchers have discussed the prediction of compressive strength of concrete based on the dynamic modulus of elasticity. Moreover, Goulias & Ali (58) have proposed a mathematical model, Equation [17], during the early investigations on RBC. In this section, the capability of this model will be investigated and compared against the proposed one based on the collected findings from the literature with the descriptive statistics shown in Table 11.

Goulias & Ali (58)
$$f_{c28} = 0.9E_d - 11.391$$
 [17]

TABLE 11. Descriptive statistics for the dataset used in the correlation between dynamic modulus of elasticity and compressive strength.

| Statistics | R _c | E _d | $f_{_{c28}}$ |
|--------------------|----------------|----------------|--------------|
| Sample size | 99 | 99 | 99 |
| Mean | 58.41 | 28.092 | 36.19 |
| Standard error | 5.02 | 0.848 | 1.44 |
| Standard deviation | 49.95 | 8.438 | 14.32 |
| Minimum | 0 | 11.3 | 10 |
| First quartile | 15.9 | 21.7 | 25.24 |
| Median | 47.5 | 28.5 | 33.85 |
| Third quartile | 80.1 | 34.65 | 45.75 |
| Maximum | 227.3 | 44.6 | 75.3 |

In fact, to produce good non-destructive models for RBC, it is suggested to use both the dynamic modulus of elasticity and the mixture's rubber content as inputs to the prediction equations. The proposed models for each specimen size are shown in Equation [18] to Equation [20]. In general, substituting zero instead of the R_c term means that the estimation is being conducted for normal concrete.

100 Cube
$$f_{c28} = 9.1\sqrt{E_d} - \frac{R_c}{6.9}$$
 [18]

150 Cube
$$f_{c28} = 6.6\sqrt{E_d} - \frac{R_c}{8}$$
 [19]

Cylinders
$$f_{c28} = 5.75\sqrt{E_d} - \frac{R_c}{12}$$
 [20]

The R^2 values are 0.797, 0.948, and 0.923 for the 100 cube, 150 cube, and cylinders models respectively. In addition, the performance of the models is shown in Figure 6. Generally, it can be seen that the models are highly capable of predicting the compressive strength of concrete with normal to high strength capacity. However, they show some deficiency in estimating values above 60 MPa, as shown in Figure 6-a and Figure 6-c, due to the lack of more experimental data to improve the prediction model. On the other hand, the residual plot in Figure 6-b depicts that the 150 cube and cylinders equations provide lower error values as compared to the 100 cube one due to the existence of a larger dataset with various rubber content in the latter case, which resulted in the more generalized model but will lower fitting rate.

A comparison between the proposed model and the one suggested by Goulias & Ali (58) is provided in Figure 7. In general, the proposed one provides significantly better matching capabilities, Figure 7-c, performance compared to the measured values, Figure 7-d, and reduced errors, Figure 7-e, compared to the Goulias & Ali (58) approach. This can be attributed to utilizing a larger dataset and introducing the R_c term in the prediction equation, which has improved the fitting performance significantly.

The error analysis of these models is given in Table 12. It can be seen that the percentage of error represented by MAPE value reached its highest of 16% in the case of 100 cube which means that the proposed models can be used reliably in estimating the compressive strength of RBC mixtures. Furthermore, the errors in the proposed method were considerably lower than these of the Goulias & Ali (58) model.

TABLE 12. Error analysis of the correlations.

| Specimen Size | Error Type | Model Performance | Goulias & Ali |
|------------------|------------|-------------------|---------------|
| 100 Cuba | RMSE | 6.55 | - |
| 100 Cube | MAPE | 15.93 | - |
| 150 Cube | RMSE | 2.40 | - |
| | MAPE | 9.45 | - |
| Cylinders | RMSE | 1.98 | 9.49 |
| | MAPE | 6.48 | 29.55 |



FIGURE 6. Comparison between the correlations of compressive strength and dynamic modulus of elasticity.

3.6. Relation between dynamic and static moduli of elasticity

another common mathematical expression in nondestructive testing is the one between dynamic and static moduli of elasticity. Such an approach was previously discussed in the literature for the case of conventional concrete (59, 60) and RBC (58). Furthermore, Habib et al. (10) observed that there is a good correlation between the static and dynamic moduli of elasticity of RBC mixtures. In this section, the applicability of previously proposed models, Equation [21] to Equation [23], to RBC will be investigated based on the dataset with the descriptive statistics shown in Table 13, and an enhanced model will be suggested.

| Lydon & Balendran (59) | $E_s = 1.25E_d - 19$ | [21] |
|------------------------|----------------------|------|
| | | |

BS 8110-2 (60) $E_s = 0.83E_d$ [22]

Goulias & Ali (58) $E_s = 1.4514E_d - 20$ [23]

TABLE 13. Descriptive statistics for the dataset used in the correlation between dynamic and static Moduli.

| Statistics | R _c | E _d | Es |
|--------------------|----------------|----------------|--------|
| Sample size | 78 | 78 | 78 |
| Mean | 48.14 | 27.644 | 24.218 |
| Standard error | 4.89 | 0.931 | 0.449 |
| Standard deviation | 43.18 | 8.219 | 3.966 |
| Minimum | 0 | 11.54 | 17.9 |
| First quartile | 15.9 | 21.7 | 21.3 |
| Median | 47.5 | 27.1 | 23.8 |
| Third quartile | 65.97 | 34.387 | 26.863 |
| Maximum | 227.3 | 44.6 | 33.2 |

Similar to the previous section, both the dynamic modulus of elasticity and the rubber content were used as inputs to the proposed prediction model, Equation [24]. Generally, the R² value of this mod-



FIGURE 7. Comparison between the correlations of compressive strength and dynamic moduli of elasticity.

el was computed as 0.77. Furthermore, the performance of the model is shown in Figure 8-a.

$$E_s = 0.35E_d - \frac{3.3R_c}{100} + 16.1$$
 [24]

A comparison between the capability of the proposed model and the previously introduced ones are shown in Figure 8. In general, the estimation model suggested in this study provides significantly better performance than the other models. Furthermore, the residuals plot, Figure 8-e, depicts that there are very high errors in the case of BS 8110-2 (60) model and Goulias & Ali (58) one as compared to the Lydon & Baledran (59) model, whereas, the proposed expression gives the lowest errors among other equations. Moreover, the suggested model achieved the best matching to the reference dataset compared to others, as seen in Figure 8-f and Figure 8-g.

The error analysis, Table 14, proves that the proposed model gives the best results among others, while the errors in Lydon & Baledran (59) were lower than both Goulias & Ali (58) model and the BS 8110-2 (60) model. On the other hand, the highest error was observed in the case of BS 8110-2 (60).

TABLE 14. Error analysis of the prediction models.

| | Model Performance | BS 8110-2 | Lydon & Balendran | Goulias & Ali |
|------|----------------------|-----------|----------------------|------------------|
| RMSE | 1.89 | 11.41 | 4.49 | 9.89 |
| MAPE | 6.14 | 41.42 | 17.05 | 36.24 |

3.7. Relation between dynamic modulus of elasticity and flexural strength

this section is intended to propose a relationship to estimate the flexural strength of RBC using its dynamic modulus of elasticity. The descriptive statistics of the dataset are shown in Table 15.

The rubber content and the dynamic modulus of elasticity were used in the estimation model given in Equation [25]. In fact, the model has an R^2 value of 0.6 and a performance, as shown in Figure 9. It can



FIGURE 8. Comparison between the correlations of static and dynamic moduli of elasticity.

| R _c | E _d | f_r |
|----------------|---|--|
| 15 | 15 | 15 |
| 65.6 | 34.98 | 5.68 |
| 11.2 | 1.57 | 0.473 |
| 43.3 | 6.07 | 1.831 |
| 0 | 24.1 | 3.185 |
| 44.1 | 30 | 3.47 |
| 50.3 | 34.11 | 6.115 |
| 88.1 | 39.84 | 7.45 |
| 132.2 | 44.6 | 8.42 |
| | R _c 15 65.6 11.2 43.3 0 44.1 50.3 88.1 132.2 | R_c E_d 15 15 65.6 34.98 11.2 1.57 43.3 6.07 0 24.1 44.1 30 50.3 34.11 88.1 39.84 132.2 44.6 |

TABLE 15. Descriptive statistics for the dataset used in the correlation between dynamic modulus of elasticity and flexural strength.

be seen from Figure 9-c that the model provides an acceptable accuracy in the prediction.

$$f_r = 0.25E_d + \frac{15.2R_c}{1000} - 4.06$$
 [25]

Moreover, the error analysis, Table 16, indicates that the model provides good results since its MAPE value is about 15.5%.

TABLE 16. Error analysis of the prediction models.

| Error Type | Model Performance |
|------------|-------------------|
| RMSE | 1.12 |
| MAPE | 15.54 |



FIGURE 9. Comparison between the correlations of flexural strength and dynamic modulus of elasticity.

4. CONCLUSIONS

This study has focused on investigating the effect of rubber aggregates including its hardened density, compressive and flexural strengths, static and dynamic moduli, damping ratio, and natural frequency of RBC using a high number of experimental observations, observe the correlation between the reduction in the compressive strength of concrete and change in the other mechanical and dynamic properties of RBC, and to propose some prediction models for this type of concrete. Overall, about 1000 experimental observations were used in the numerical investigations. This study proposed several prediction models, including some relationships for nondestructive testing. It was shown that these equations could be used reliably to predict the hardened density, mechanical, and dynamic properties of RBC. Moreover, based on the comparative study that was conducted in some sections to evaluate the capabilities of the proposed models against the available ones, it was clear that the suggested estimation methods provide better performance in the case of RBC. Further research is still

needed in the field of RBC to investigate the influence of mixing different types of recycled aggerates with rubber particles green concrete with an enhanced vibration behavior and provide a solid understanding of the performance of this material when used in structural applications especially in earthquake-prone counties.

AUTHOR CONTRIBUTIONS:

Conceptualization: A. Habib, U. Yildirm. Methodology: A. Habib, U. Yildirm. Formal analysis: A. Habib. Writing, original draft: A. Habib. Writing, review & editing: U. Yildirm. Visualization: A. Habib.

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